for a Gurevich medium

$$f = 2\frac{c_\infty}{\tau}\frac{1}{A}\frac{a'}{1-a'}(1 - A\ln a')^{1/2},\ g = (1 - A\ln a')^{-1/2}.$$

Here A is the ratio of the shear elastic modulus to the relaxation one, and a' is the ratio of minimum to maximum relaxation times.

Comparison of (A.5) and (A.4) shows that at distances $x \gg c_\infty\tau$ the maximum of I(t, x) is displaced with velocity c_0, and is determined by the expansion of $K_2(p)$ near the point p = 0. We note that for $\tau \to 0$ there exists for the representation (A.5) a limiting transition to the case of an ideal elastic medium $I(t, x) = \delta(t - x/c_\infty)$, since $f \sim \tau^{-1}$.

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STUDY OF ELASTOPLASTIC DEFORMATION FOR CYLINDRICAL SHELLS WITH AXIAL SHOCK LOADING

A. I. Abakumov, G. A. Kvaskov,
S. A. Novikov, V. A. Sinitsyn,
and A. A. Uchaev

UDC 620.178.7

There is considerable practical interest in studying the dynamic stability of cylindrical shells under the action of axial intense shock loads. A shell is assumed to be dynamically stable if its movement is not accompanied by buckling, i.e., it is constrained. The nature of loss of stability for a cylindrical shell is determined mainly by its relative thickness h/R (h is shell thickness, R is central surface radius). For relatively thin shells with h/R < 1/100 elastic buckling is normally considered when loss of stability occurs with formation of rhombic hollows, and shell deflection as a result of sudden popping. With an increase in relative shell thickness plastic buckling is observed during its axial compression. Plastic loss of stability is characterized by the fact that the shell may demonstrate marked resistance to buckling. In the initial stage of deformation with plastic buckling there is almost always axisymmetrical loss of stability in the form of an annular fold caused by the effect of boundary conditions at the shell edges. With further axial compression the shell continues to lose stability in axisymmetrical shape or it may change over to an asymmetrical form of loss of stability. It was shown by experiment in [1] that the form of loss

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 150-153, May-June, 1988. Original article submitted February 3, 1987.

0021-8944/88/2903-0450$12.50

TABLE 1

Test No.	Striker weight, g	Initial velocity, m/sec	Shell crumpling, %
1	245	70	25
2	390	70	39
3 *	390	70	45
4	730	65	62

*Test without filler.

of stability depends on the relative thickness, in particular for aluminum shells a changeover from axisymmetrical to asymmetrical loss of stability is observed with $h/R < 1/5$. Analysis of the mechanism for loss of stability indicates that for formation of the axisymmetrical form with $h/R < 1/5$ it is necessary that compressive axial stresses are balanced by internal pressure with a capacity to block development of asymmetry in the shell during flexure. Internal pressure in the shell during deformation may be created as a result of filling its cavity with compressible (e.g., porous) material. Attainment of a stable axisymmetrical form of loss of stability makes it possible in order to describe the behavior of a cylindrical shell with shock loading to use a procedure for calculating elastoplastic deformation of shells of rotation with axisymmetrical dynamic loading. The form of cylindrical shells after axial shock compression with asymmetrical (a is test 3, see Table 1) and axisymmetrical (b, with the presence of internal pressure test 2 c, is the calculation) forms of loss of stability is shown in Fig. 1.

Experiments were carried out with cylindrical shells made of aluminum alloy AMg6 prepared from standard tubes. Shell geometric dimensions were $R = 9.3$ mm, $h = 1.5$ mm, length $\ell = 80$ mm. Shell loading was accomplished by impact of a metal plate-striker over the end. The other end of the shell rested on an immovable support, i.e., a measuring rod (dynamometer). Compressive force measurement in each test, as in [1], was carried out by means of strain gauges glued to the dynamometer. In order to accomplish a stable axisymmetrical form of deformation the internal volume of the shell was filled with a porous material with density $\rho = 0.2$ g/cm^3. The main shell loading parameters are presented in Table 1.

A number of works [2-7] have been devoted to the question of numerical description of axisymmetrical elastoplastic shell buckling in which with shock loading the process of shell behavior is only considered in the initial stage of forming folds. The complex nature of solving the problem with description for fold formation means that it is impossible to consider contact forces in the zone of a fold and the stress–strain state through the shell thickness. If as a final result we only take integral characteristics such as shell shape change and support reaction, then the requirement for quite accurate knowledge of the stress-strain state in the fold zone disappears. This makes it possible to describe fold formation approximately by using the condition of reciprocal nonpenetration of shell elements. Proceeding from this a model was suggested for fold formation in cylindrical shells with shock loading. At the heart of the model is a procedure [6] based on Timoshenko-type shell theory, plastic flow theory, and a variation-difference method for numerical solution. Large deflections are considered by stepwise reconstruction of the position of the central surface of the shell. In the model starting from the instant of time at which a deflection occurs in the shell exceeding its thickness a control is introduced for the nature of buckling. The control process includes the following:

a) determination of calculation nodal points for the shell falling at the tip or hollow of the folds formed;

b) the phenomenon of possible closing up of part of the shell surface (internal and external) with formation of a fold. According to experiments the number of folds analyzed should not be less than two from both edges of the shell. The process of closing up for a given load first of all ceases at that fold which is located closer to the shell edge from the direction of the immovable support.

In developing a calculation for closing up of some part of a fold, a conversion is carried out to longitudinal velocity for each pair of points in contact by averaging their velocities at the instant of contact. For points in contact with the striker or support the

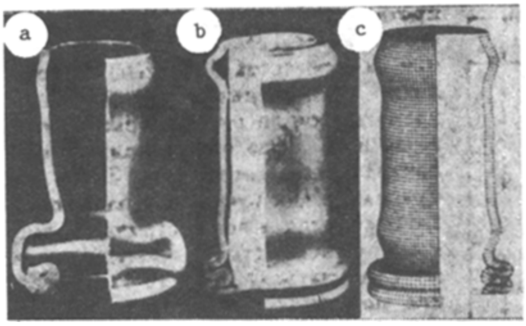

Fig. 1

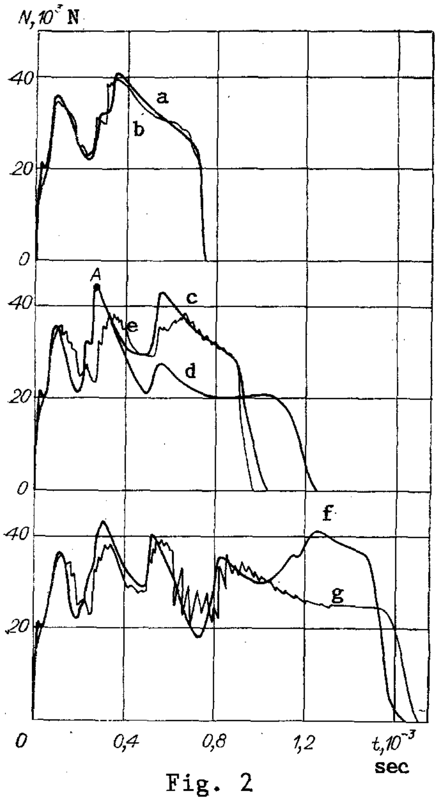

Fig. 2

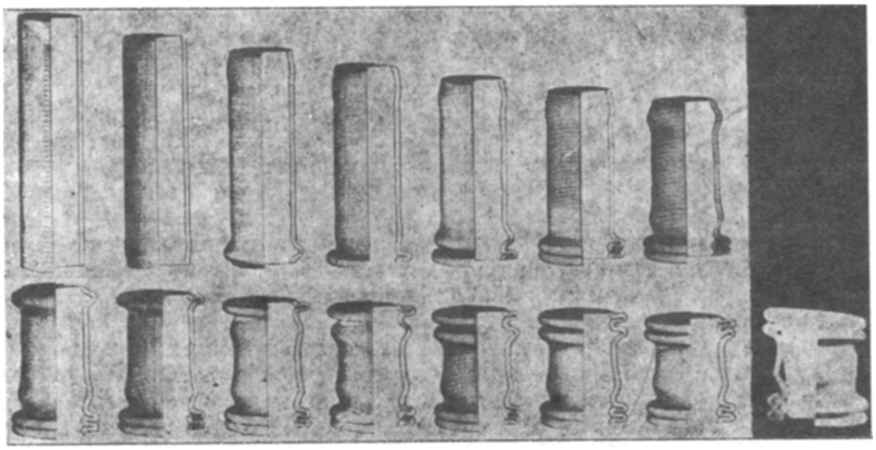

Fig. 3

velocity is compared with striker or support velocity, respectively. The retardation force for the striker is taken to be equal to the longitudinal force of resistance to shell deformation in a zone distant from the region of points which are in contact.

It should be noted that calculated data are in sufficiently good agreement with experimental results when boundary conditions for the shell edges are conditions for their jointed fixing with an immovable support and striker.

Comparison of calculated values with experimental values was carried out according to two parameters: time dependence of compressive force $N = N(t)$ and residual shape of shell. Given in Fig. 2 are experimental (a, c, d, f for tests 1-4) and calculated (b, e, g for loading conditions in tests, 1, 2, 4) of relationship $N = N(t)$.

Use of a porous filler for creating internal pressure in a shell promoted axisymmetrical buckling during the whole process of axial compression (Fig. 3). Conformity of relationships $N = N(t)$ for shells with a filler and without it is observed up to point A (Fig. 2, curves c and d) when buckling in shells is axisymmetrical. Then buckling of a shell without a filler changes into an asymmetrical form and relationships $N = N(t)$ start to differ. The effect of a porous filler is not taken into account in calculations and $N = N(t)$ corresponded to the longitudinal forms realized in the shell in the zone adjacent to folds formed from the direction of the immovable support.

The good agreement can be seen in Fig. 2 of calculations and experiments for relationship N = N(t) up to quite large values of shell compression (≈40%) with which the given filler still does not have a marked effect on longitudinal force realized in the shell. The calculated nature of shape change for a cylindrical shell at different instants of time (after 100 μsec) for test No. 4 is given in Fig. 3. By comparing Fig. 3 with Fig. 2f over time it is possible to note that the increase in N = N(t) up to a critical value is observed with deflections exceeding the shell thickness, and a drop is observed with intense fold formation.

Comparison of calculated results with experimental data shows quite good agreement both for residual shell shape (see Figs. 1b, c and Fig. 3), and for the relationship N = N(t) (see Fig. 2), which points to the efficiency of the model suggested in describing shock compression of cylindrical shells of moderate thickness (h/R = 1/10...1/5).

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PLASTIC MODELS IN PROBLEMS OF ELASTIC DEFORMATION OF ROLLED SHELLS

S. V. Lavrikov and A. F. Revuzhenko

UDC 539.3

1. The questions considered in this work arose from the following considerations. We refer to classical solution of the Lamé problem for a thick-walled cylindrical tube. In view of axial symmetry for the problem tangential stresses are absent: $\sigma_{r\theta} = 0$ (r and θ are polar coordinates). This means that if an arbitrary number of cuts is made in the tube over the circumference r = const, then these cuts do not impinge on the operation of the structure. Consequently, the cross section of the tube may be represented by a collection of thin individual rings mounted close to each other; rings operate so that conditions at contacts between them do not affect the operation of the whole structure. As is well known, in this scheme the material is loaded very unevenly, and if the external radius of the tube exceeds the internal radius by more than a factor of three to four then a further increase in tube thickness has practically no effect on the change over of the inner region into a plastic condition (failure). Therefore, an idea occurs naturally: is it possible to organize the work of elastic rings in such a way that external friction forces are mobilized between them which would contribute to "resisting" external pressure. We cut up rings over a certain radius and glue them together with displacement by one pitch (Fig. 1). The structure obtained differs in principle from the previous one. It might be expected that as a result of slippage of layers it will be possible to include in the operation material distant from the inner boundary, and consequently to distribute the applied load more uniformly thus incresing the supporting capacity of the structure.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 153-159, May-June, 1988. Original article submitted March 18, 1987.

0021-8944/88/2903-0453$12.50